Programming Distributed Systems

03 Causality and Vector clocks

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Motivation

- Causality is fundamental to many problems occurring in distributed computing
- *Examples*: Determining a consistent recovery point, detecting race conditions, exploitation of parallelism
- The happens-before relation of events is often also called *causality relation* [1].

An event e may causally affect another event e' if and only if $e \to e'$.

- The happens-before order $\to$ indicates only *potential* causal relationship.
- Tracking whether an event indeed is a cause of another event is much more involved and requires more complex dependency analysis.

Overview

- Causal Broadcast
- Causality Tracking with Vector clocks
- Causal Broadcast revisited

(Reliable) Causal Broadcast (RCO): Specification

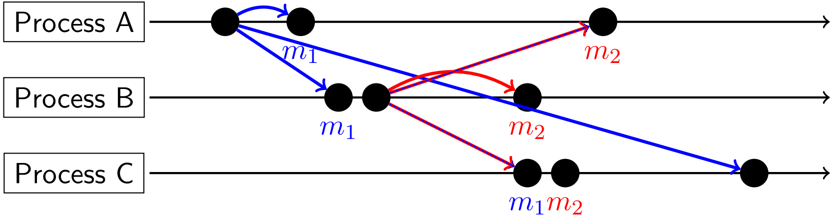
- *RB1 - RB4* from reliable broadcast
- *CB (Causal delivery):* No process p delivers a message m' unless p has already delivered every message m such that $m \rightarrow m'$.

Causal Broadcast (RCO): Algorithm 1 (No-waiting)

```
State:
  delivered //set of messages ids that were already rcoDelivered
  past // ordered set that it has rco-Broadcast or rco-Delivered
Upon Init do:
  delivered <- $\emptyset$;
  past      <- $\emptyset$;
Upon rco-Broadcast(m) do
  $m_{id}$ <- generateUniqueID(m);
  trigger rb-Broadcast([$m_{id}$ , past, m]);
  past <- past $\cup$ {(self, $m_{id}$, m)}; // ordered after prior entries

Upon rb-Deliver(p, [$m_{id}$, $past_m$, m]) do
  if ( $m_{id} \notin$ delivered ) then
    forall ($s_n$, $n_{id}$, n) in $past_m$ do // deterministic order!
      if ($n_{id} \notin$ delivered ) then
        trigger rco-Deliver($s_n$, n);
        delivered <- delivered $\cup$ {$n_{id}$};
        past      <- past $\cup$ {($s_n$, $n_{id}$, n)};
    trigger rco-Deliver(p, m);
    delivered <- delivered $\cup$ {$m_{id}$};
    past      <- past $\cup$ {(p, $m_{id}$, m)};
```

Causal Broadcast: Scenario 1



Remarks

- Message id's could be reused for RB broadcast
- $past_m$ of a message includes all messages that causally precede m
- Message from causal past of m are delivered before message m
- Size of messages grows linearly with every message that is broadcast since it includes the complete causal past
- *Idea*: Garbage collect the causal past
  - If we know when a process fails (i.e., under the Fail-stop model), we can remove messages from the causal past
  - When a process rb-Delivers a message m, it rb-Broadcasts an acknowledgement message to all other processes
  - When an acknowledgement for message m has been rbDelivered by all correct processes, m is removed from $past$
  - N^2 additional ack messages for each data message
  - Typically, acknowledgements are grouped and processed in batch mode

Causality tracking with Vector clocks

Causal Histories

- We here distinguish three types of events occurring in a process:
  - Send events
  - Receive events
  - Local / internal events
- Let E_i denote the set of events occurring at process p_i and E the set of all executed events:

$$E = E_1 \cup \cdots \cup E_n$$

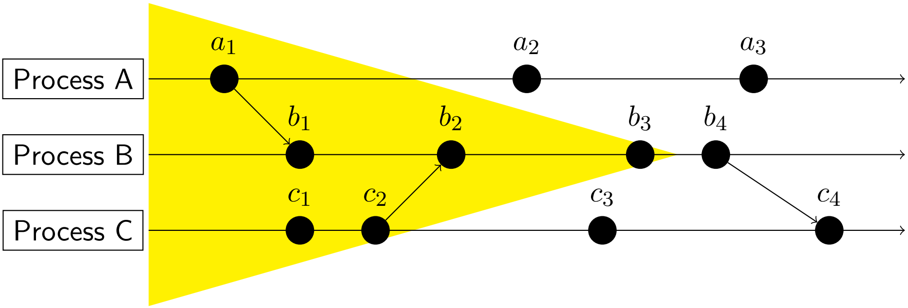
- The *causal history* of an event $e \in E$ is defined as

$$C(e) = \{e' \in E \mid e' \to e\} \cup \{e\}$$

- Note: Just a different representation of happens-before:

$$e' \to e \quad \Leftrightarrow \quad e' \neq e \wedge e' \in C(e)$$

Example: Causal history of b_3



$$C(b_3) = \{a_1, b_1, b_2, b_3, c_1, c_2\}$$

Tracking causal histories

Each process p_i stores current causal history as set of events C_i.

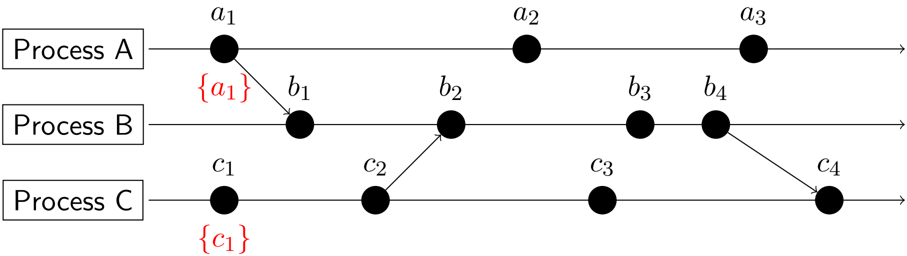
- Initially, $C_i \leftarrow \emptyset$
- On each local event e at process p_i, the event is added to the set:

$$C_i \leftarrow C_i \cup \{e\}$$

- On sending a message m, p_i updates C_i as for a local event and attaches the new value of C_i to m.
- On receiving message m with causal history $C(m)$, p_i updates C as for a local event. Next, p_i adds the causal history from C(m):

$$C_i \leftarrow C_i \cup C(m)$$

Example: Causal histories



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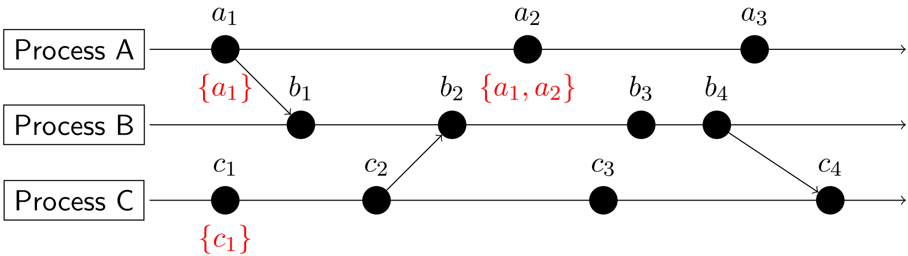
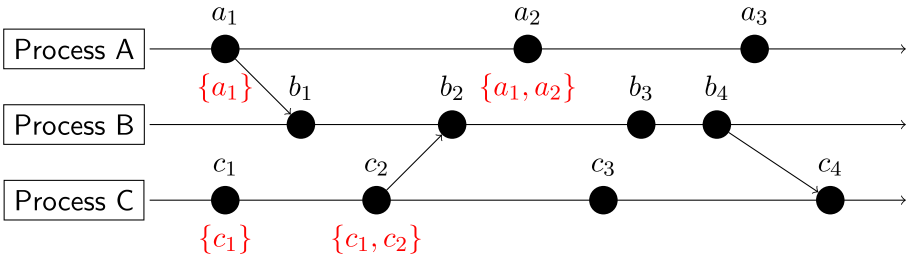


Example: Causal histories



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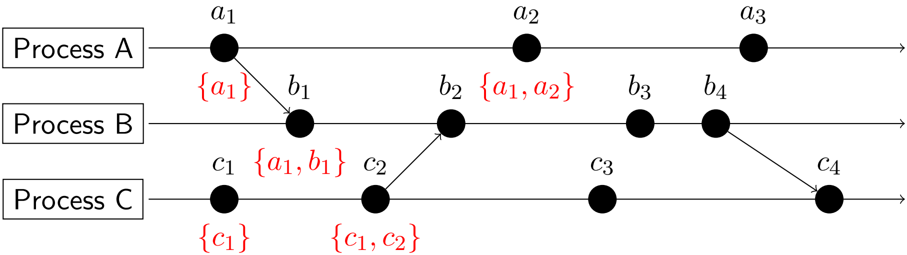

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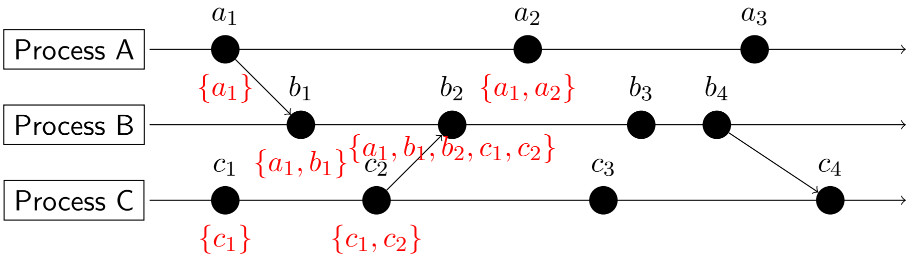

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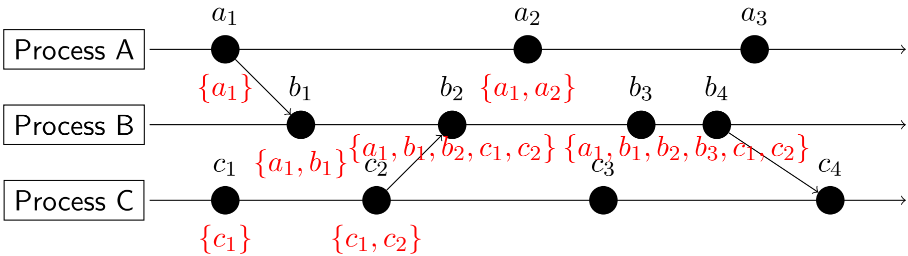

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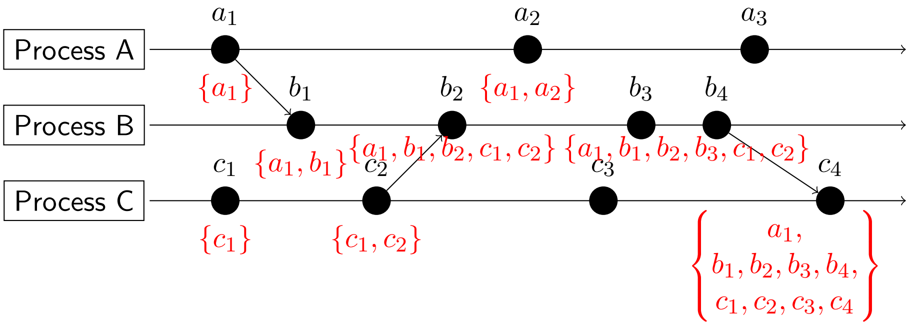

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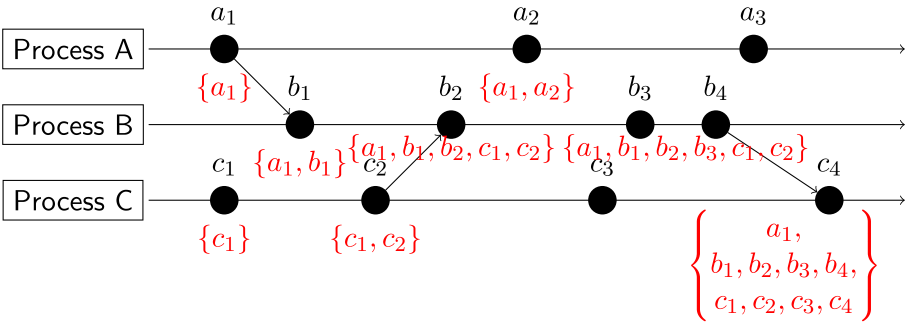
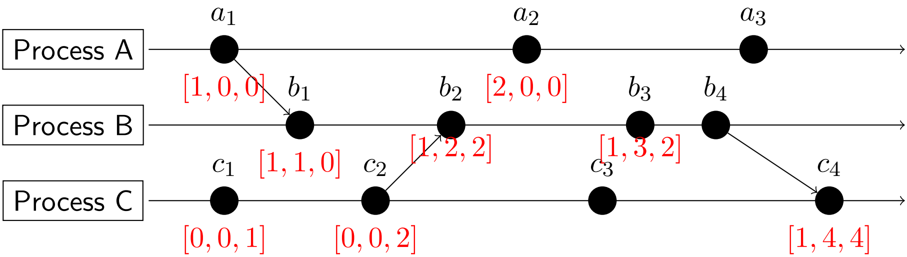


Can we represent causal histories more efficiently?

Example: Efficient representation of causal histories



Efficient representation of causal histories

- Vector clock $V(e)$ as efficient representation of $C(e)$.
- Vector clock is a mapping from processes to natural numbers:
  - Example: $[p_1 \mapsto 3, p_2 \mapsto 4, p_3 \mapsto 1]$
  - If processes are numbered $1, \ldots, n$, this mapping can be represented as a vector, e.g. [3, 4, 1]
  - Intuitively: $p_1 \mapsto 3$ means "observed 3 events from process p_1"

Formal Construction

- Assume processes are numbered 1, . . . , n
- Let $E_k = \{e_{k_1}, e_{k_2}, \dots\}$ be the events of process k
  - Totally ordered: $e_{k_1} \to e_{k_2}, e_{k_2} \to e_{k_3}, \dots$
- Let $C(e)[k] = C(e) \cap E_k$ denote the projection of $C(E)$ on process k.

$$C(e) = C(e)[1] \cup \dots \cup C(e)[n]$$

- Now, if $e_{k_j} \in C(e)[k]$, then by definition it holds that $e_{k_1}, \dots, e_{k_j} \in C(e)[k]$
- The set $C(e)[k]$ is thus sufficiently characterized by the largest index of its events, i.e. its cardinality!
- Summarize $C(e)$ by an n-dimensional vector $V(e)$ such that for $k = 1, \dots, n$:

$$V(e)[k] = |C(e)[k]|$$

Note: Both representations are lattices with a lower bound

Operator	Causal history	Vector clock
$\bot$	$\emptyset$	$\lambda i.\ 0$
$A \leq B$	$A \subseteq B$	$\forall i.\ A[i] \leq B[i]$
$A \geq B$	$A \supseteq B$	$\forall i.\ A[i] \geq B[i]$
$A \sqcup B$	$A \cup B$	$\lambda i.\ max(A[i], B[i])$
$A \sqcap B$	$A \cap B$	$\lambda i.\ min(A[i], B[i])$

- $\bot$: bottom, or smallest element
- $A \sqcup B$: least upper bound, or join, or supremum
- $A \sqcap B$: greatest lower bound, or meet, or infimum

Tracking causal histories

Each process p_i stores current causal history as set of events C_i.

- Initially, $C_i \leftarrow \emptyset$
- On each local event e at process p_i, the event is added to the set: $C_i \leftarrow C_i \cup \{e\}$
- On sending a message m, p_i updates C_i as for a local event and attaches the new value of C_i to m.
- On receiving message m with causal history $C(m)$, p_i updates C_i as for a local event. Next, p_i adds the causal history from C(m):

$$C_i \leftarrow C_i \cup C(m)$$

Tracking causal histories

Each process p_i stores current causal history as set of events C_i.

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- On sending a message m, p_i updates C_i as for a local event and attaches the new value of C_i to m.
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$$C_i \leftarrow C_i \sqcup C(m)$$

Vector time

Each process p_i stores current causal history as a vector clock V_i.

- Initially, $V_i[k] \leftarrow \bot$
- On each local event, process p_i increments its own entry in V_i as follows: $V_i[i] \leftarrow V_i[i] + 1$
- On sending a message m, p_i updates V_i as for a local event and attaches new value of V_i to m.
- On receiving message m with vector time $V(m)$, p_i increments its own entry as for a local event. Next, p_i updates its current V_i by joining $V(m)$ and V_i:

$$V_i \leftarrow V_i[k] \sqcup V(m)$$

Relating vector times

Let u, v denote time vectors. We say that

- $u \leq v$ iff $u[k] \leq u[k]$ for $k = 1, \ldots, n$
- $u < v$ iff $u \leq v$ and $u \neq v$
- $u \parallel v$ iff neither $u \leq v$ nor $v \leq u$

For two events e and e', it holds that $e \rightarrow e' \quad \Leftrightarrow \quad V(e) < V(e')$

- Proof: By construction.

How does vector time relate to Lamport timestamps?

- Both are logical clocks, counting events.
- Lamport time (and real time) are insufficient to characterize causality and can't be used to prove that events are not causally related

Causal Broadcast (RCO): Algorithm 2 (Waiting)

```
State:
  pending //set of messages that cannot be delivered yet
  VC // vector clock
Upon Init do:
  pending <- $\emptyset$;
  forall $p_i \in \Pi$ do: VC[$p_i$] <- 0;

Upon rco-Broadcast(m) do
  trigger rco-Deliver(self, m);
  trigger rb-Broadcast(VC, m);
  VC[self] <- VC[self] + 1;

Upon rb-Deliver(p, $VC_m$, m) do
  if ( p $\neq$ self ) then
    pending <- pending $\cup$ {(p, $VC_m$, m)};
    while exists (q, $VC_{m_q}$, $m_q$) $\in$ pending, such that VC $\geq$ $VC_{m_q}$ do
        pending <- pending \ {(q, $VC_{m_q}$, $m_q$)};
        trigger rco-Deliver(q, $m_q$);
        VC[q] <- VC[q] + 1;
```

Causal Broadcast (RCO): Algorithm 2 (Waiting)

```
State:
  pending //set of messages that cannot be delivered yet
  VC // vector clock
Upon Init do:
  pending <- $\emptyset$;
  forall $p_i \in \Pi$ do: VC[$p_i$] <- 0;

Upon rco-Broadcast(m) do
  trigger rco-Deliver(self, m);
  trigger rb-Broadcast(VC, m);
  VC[self] <- VC[self] + 1;

Upon rb-Deliver(p, $VC_m$, m) do
  if ( p $\neq$ self ) then
    pending <- pending $\cup$ {(p, $VC_m$, m)};
    while exists (q, $VC_{m_q}$, $m_q$) $\in$ pending, such that VC $\geq$ $VC_{m_q}$ do
        pending <- pending \ {(q, $VC_{m_q}$, $m_q$)};
        trigger rco-Deliver(q, $m_q$);
        VC[q] <- VC[q] + 1;
```

Limitations of Causal Broadcast

- Processes can observe messages in different order!

Example: Replicated database handling bank accounts

- Initially, account A holds 1000 Euro.
- User deposits 150 Euro, triggers broadcast of message
  $m_1 =$ `'add 150 Euro to A'`
- Concurrently, bank initiates broadcast of message
  $m_2 =$ `'add 2% interest to A'`
- Diverging state!

⇒ Later lecture: Atomic broadcast!

Summary

- Causality important for many scenarios
- Causality not always sufficient
- Vector clocks:
    - Efficient representation of causal histories / happens-before
    - How many events from which process?
- Causal broadcast: Use vector clocks to deliver in causal order

Further reading I

[1] Reinhard Schwarz und Friedemann Mattern. “Detecting Causal Relationships in Distributed Computations: In Search of the Holy Grail”. In: *Distributed Computing* 7.3 (1994), S. 149–174. DOI: 10.1007/BF02277859. URL: https://doi.org/10.1007/BF02277859.