

**Lecture: Replication and Consistency**  
**Exercise Sheet 2**

<https://pl.cs.uni-kl.de/homepage/de/teaching/ws19/rac/>

## 1 Quiescent consistency

Quiescent consistency is a weaker variant of linearizability. It also requires that that method calls should appear to happen in a one-at-a-time, sequential order. But under quiescent consistency, only method calls *separated by a period of quiescence* appear to take effect in their real-time order.

Give an example of an execution that is quiescently consistent but not sequentially consistent, and another that is sequentially consistent but not quiescently consistent.

## 2 Atomic Integers

The `AtomicInteger` class (in the `java.util.concurrent.atomic` package) is a container for an integer value. One of its methods is `boolean compareAndSet(int expect, int update)`. This method compares the object's current value to `expect`. If the values are equal, then it atomically replaces the object's value with `update` and returns `true`. Otherwise, it leaves the object's value unchanged, and returns `false`. This class also provides `int get()` which returns the object's actual value.

Consider the FIFO queue implementation:

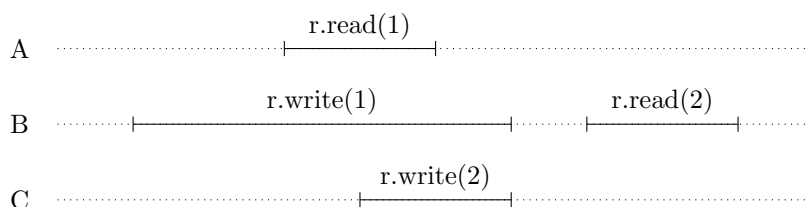
```
class IQueue<T> {
    AtomicInteger head = new AtomicInteger(0);
    AtomicInteger tail = new AtomicInteger(0);
    T[] items = (T[]) new Object[Integer.MAX_VALUE];
    public void enq(T x) {
        int slot ;
        do {
            slot = tail.get();
        } while (! tail.compareAndSet(slot, slot+1));
        items [slot] = x;
    }
    public T deq() throws EmptyException {
        T value;
        int slot;
        do {
            slot = head.get();
            value = items[slot];
            if (value == null)
                throw new EmptyException();
        } while (! head.compareAndSet(slot, slot+1));
        return value;
    }
}
```

It stores its items in an array `items`, which, for simplicity, we will assume has unbounded size. It has two `AtomicInteger` fields: `tail` is the index of the next slot from which to remove an item, and `head` is the index of the next slot in which to place an item. Give an example showing that this implementation is not linearizable.

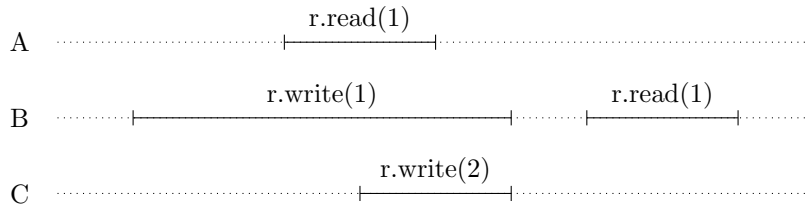
## 3 Classifying histories

For each of the histories shown, are they quiescently consistent? Are they sequentially consistent? Or linearizable? Justify your answer.

History 1:



History 2:



## 4 Strange methods....

Consider the following rather unusual implementation of a method `m`. In every history, the  $i^{\text{th}}$  time a thread calls `m`, the call returns after  $2^i$  steps. Is this method wait-free, bounded wait-free, or neither?

## 5 Back to Peterson

Does Peterson's two-thread mutual exclusion algorithm work if we replace shared atomic registers with regular registers?

## 6 Atomic Registers

You learn that your competitor, the Acme Atomic Register Company, has developed a way to use Boolean (single-bit) atomic registers to construct an efficient write-once single-reader single-writer atomic register for an  $N$ -valued integer. Through your spies, you acquire the code fragment shown below, which is unfortunately missing the code for `read()`. Your job is to devise a `read()` method that works for this class, and to justify (informally) why it works. (Remember that the register is write-once, meaning that your read will overlap at most one write.)

```
class AcmeRegister implements Register {
    // N is the total number of threads

    // Atomic multi-reader single-writer registers
    private BoolRegister [] b = new BoolMRSWRegister[3 * N];
    public void write(int x) {
        boolean[] v = intToBooleanArray(x);
        // copy v[i] to b[i] in ascending order of i
        for (int i = 0; i < N; i++)
            b[i].write(v[i]);
        // copy v[ i ] to b[N+i] in ascending order of i
        for (int i = 0; i < N; i++)
            b[N+i].write(v[ i ]);
        // copy v[ i ] to b[2N+i] in ascending order of i
        for (int i = 0; i < N; i++)
            b[(2*N)+i].write(v[i]);
    }
    public int read() {
        // missing code
    }
}
```

## 7 Variations on the Bakery Algorithm

Define a wraparound register that has the property that there is a value  $v$  such that adding 1 to  $v$  yields 0, not  $v + 1$ . If we replace the Bakery algorithm's shared variables with a (a) regular, (a) safe or (b) wraparound registers, then does it still satisfy (1) mutual exclusion, (2) FIFO ordering?